HYPOTHESIS TESTING • MTH107

Class R FAQ

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1-Sample Z-Test

H_0: $\mu = \mu_0$ (where μ_0 = specific value)

Statistic: $\bar{x}$ **Test Statistic:** $Z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$ **Conf. Region:** $\bar{x} + Z^* \frac{\sigma}{\sqrt{n}}$

Assumptions: 1) σ is known
2) $n \geq 30$, $n \geq 15$ and population not strongly skewed, OR population is normal

R: z.test()

1-Sample t-Test

H_0: $\mu = \mu_0$ (where μ_0 = specific value)

Statistic: $\bar{x}$ **Test Statistic:** $t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ **Conf. Region:** $\bar{x} + t^* \frac{s}{\sqrt{n}}$ **df:** n-1

Assumptions: 1) σ is UNknown,
2) $n \geq 40$, $n \geq 15$ & histogram not strongly skewed, OR histogram is normal

R: t.test()

2-Sample t-Test

H_0: $\mu_1 = \mu_2$ **Statistic:** $\bar{x}_1 - \bar{x}_2$

Test Statistic: $t = \frac{(\bar{x}_1-\bar{x}_2)-0}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

Conf. Region: $(\bar{x}_1 - \bar{x}_2) + t^* \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ **df:** $n_1 + n_2 - 2$

Assumptions:
1) Individuals in groups are independent
2) $n_1+n_2 \geq 40$, $n_1+n_2 \geq 15$ and both histograms are not strongly skewed, OR both histograms are normal
3) Group population variances are equal (use Levene's Test)

R: t.test(), levenesTest()

Choosing a Hypothesis Test

1. If response variable is QUANTITATIVE, GOTO 2; if CATEGORICAL, GOTO 5.

Quantitative Response
2. If 1 group/population, GOTO 3; if 2 or more groups/populations, GOTO 4.
3. If σ is KNOWN, then 1-Sample Z; If σ is UNKNOWN, then 1-Sample t.
4. If individuals are INDEPENDENT between groups, then 2-Sample t; otherwise, Paired t.

Categorical Response
5. If 1 group, then Goodness-of-Fit; if 2 or more groups, then Chi-Square.

Making a Decision about H_0

If p-value < α, then REJECT H_0., otherwise DNR H_0.

Chi-Square Test

H_0: "Distribution of individuals into response levels is the same for all groups"
H_A: "Distribution of individuals into response levels is NOT the same for all groups"

Statistic: Observed frequency table

Test Statistic: $\chi^2 = \sum \frac{(\text{Observed}-\text{Expected})^2}{\text{Expected}}$ **df:** (rows-1)(columns-1)

Assumptions: $\geq$5 in each cell of the EXPECTED table

R: xtabs(), matrix(), chisq.test(), percTable()

Goodness-of-Fit Test

H_0: "Distribution of individuals into response levels follows the theoretical distribution"
H_A: "Distribution of individuals into response levels does NOT follow the theoretical distribution"

Statistic: Observed frequency table

Test Statistic: $\chi^2 = \sum \frac{(\text{Observed}-\text{Expected})^2}{\text{Expected}}$ **df:** cells-1

Assumptions: $\geq$5 in each cell of the EXPECTED table

R: xtabs(), c(), chisq.test(), percTable(), chiGOF()

11 Steps for Any Hypothesis Test

1) State the rejection criterion (α)
2) State the null & alternative hypotheses and define the parameter(s)
3) Determine which test to perform – Explain!
4) Collect the data (address type of study and randomization)
5) Check all necessary assumption(s)
6) Calculate the appropriate statistic(s)
7) Calculate the appropriate test statistic
8) Calculate the p-value
9) State your rejection decision
10) Summarize your findings in terms of the problem
11) Compute and interpret a **100(1-α)%** *confidence region* for parameter